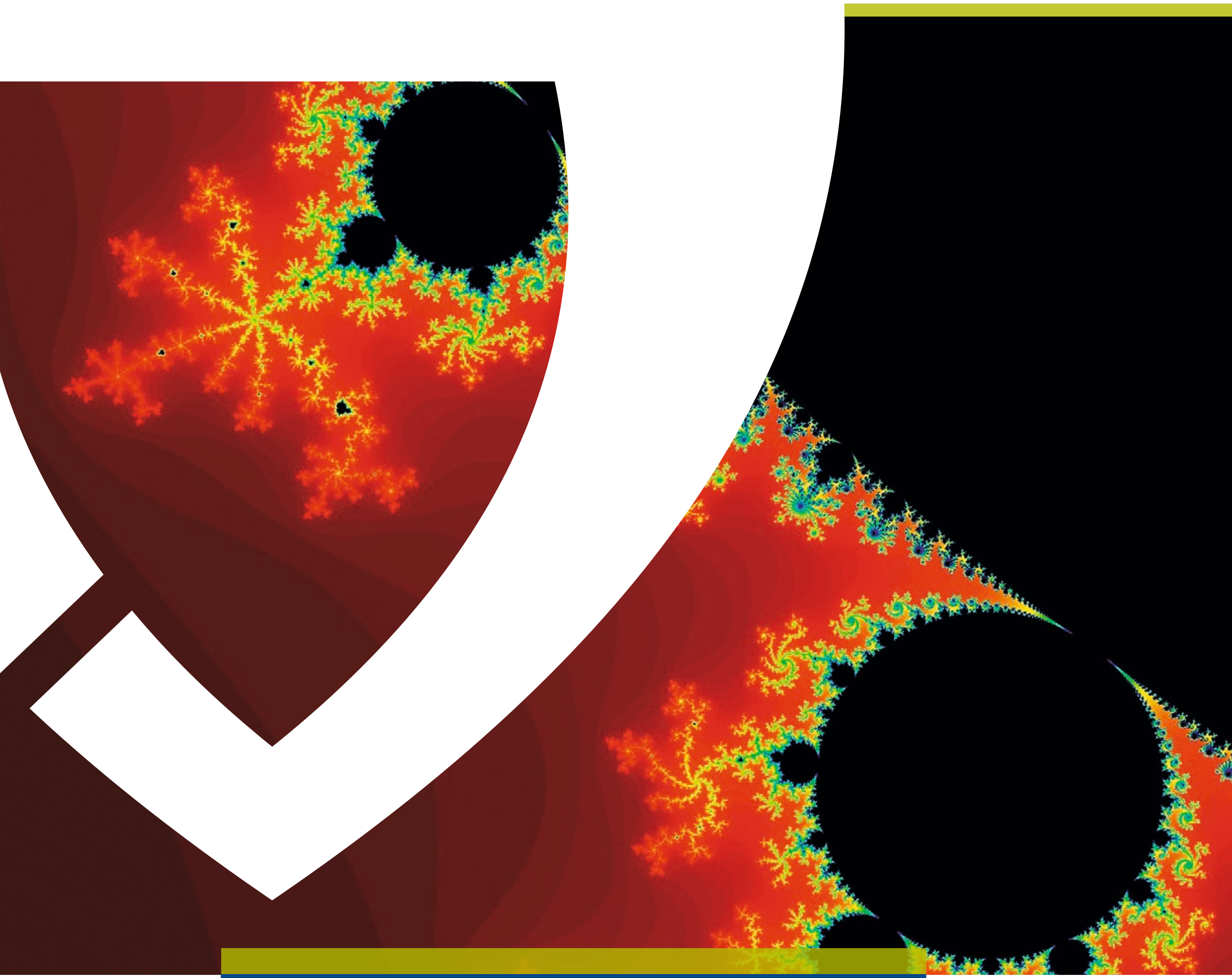
JACOBS UNIVERSITY



School of Engineering and Science

Mathematical Sciences

Graduate Program

Contents

1 Concept

1.1 General Description

The graduate program in Mathematical Sciences is part of the graduate school associated to the ”Mathematics, Modeling and Computing (MAMOC)” research center.

The Mathematical Sciences Graduate Program at Jacobs University offers the opportunity for graduate study in pure, applied and computational mathematics, as well as in mathematical physics. The program leads to a *doctorate degree (PhD)*; a *Master's degree (MSc)* may be obtained as well.
This is an “integrated PhD program” which accepts students holding a Bachelor degree, as well as more advanced students. An early beginning has the advantage that students can spend their first semesters in the program exploring research areas and meeting possible advisors before having to finalize their choice, thus making better informed decisions. More advanced students are admitted at a level compatible with their previous education.
The initial part of the program involves a broad education in mathematical science, followed by a choice of advanced courses, seminars, and research activities leading to a dissertation.
Graduate students at Jacobs University are viewed as professionals. From early on, they are integrated into the faculty's international research collaborations, they participate at international research conferences or in thematic research programs—a head start into a successful career in academia or industry.

1.2 Program Overview and Duration

Doctor of Philosophy (PhD) Students entering the graduate program with a Bachelor degree are required to complete successfully up to three semesters of coursework and a qualifying exam before progressing to the PhD dissertation. The program generally takes up to five years after the BSc degree. A separate MSc thesis is not required for students working towards a PhD degree, but students have the option to earn a separate Master's degree en route.
Students holding a Master's degree (or equivalent) typically need no more than three years until completion of their PhD degree.

Master of Science (MSc) The MSc degree requires up to three semesters of full-time coursework and one semester to produce a Master's thesis.

1.3 Interactions

Members of the Graduate Program in Mathematical Sciences interact with many faculty members and programs within the School of Engineering and Science, within Jacobs University at large, and with researchers worldwide. In particular, our weekly mathematics colloquium brings in leading mathematicians from Europe and overseas in all areas of mathematical sciences, in addition to the regular research contacts of our faculty members.
Moreover, graduate students with interests in applied, numerical, or computational mathematics are supported by Jacobs University's Computational Laboratory for Analysis, Modeling, and Visualization (CLAMV). CLAMV is equipped with advanced graphics workstations, a

Linux cluster, a Sun Fire compute server, and has access to the Northern German supercomputing network. Jacobs University offers many opportunities for interaction with researchers in other fields—including geophysics, astrophysics, computer science, physics, psychology, neurosciences, and social sciences—whose work involves mathematical modeling and computation.
Graduate students with interests in Mathematical Physics can benefit from the course offerings, seminars and research activities of the Astroparticle Physics Graduate Program at Jacobs University. Traditionally there has been a strong cross-fertilization between mathematics and physics. Mathematics provides the language and forms the foundation of modern physics. Physics has inspired many important developments in mathematics. More than ever this is true today. Graduate students who want to do research in modern mathematical or theoretical physics need a strong mathematical background as it is provided in our graduate program.

1.4 Career Options

The graduate program in mathematical sciences at Jacobs University is designed to equip students with the necessary tools and scientific maturity to embark on a research career in academia or industry. Due to the central role of mathematics in science, there is a never ceasing demand for mathematicians in academia worldwide. Universities and colleges offer tenure-track and tenured positions to PhDs; certain positions are more focused on research and others more on teaching. Graduates in mathematical sciences are well sought after by non-academic employers. Consequently, mathematicians enjoy a large choice of well-regarded jobs outside of the university world, for example in research and development, finance, banking, and management.

2 Structure of the Program

Graduate education at Jacobs University is governed by the appropriate policies. Additional program specific rules are described below.

2.1 Initial Academic Advisor

Every incoming graduate student is assigned an initial academic advisor prior to coming to Jacobs University. The initial advisor guides the graduate students through the program, monitors his or her progress, and helps him or her select a PhD advisor.

2.2 Study Plan: Integrated PhD Program

The following study plan is the default variant for students entering with a BSc degree. Faster progress is always possible.

Semester	Coursework	Research	Additional Information
1–2	3 courses, 1 seminar		
3	3 courses, 1 seminar	Preliminary work	Qualifying exam must be completed at the beginning of the 4th semester
4	1 course, 1 seminar	PhD research proposal	PhD proposal must be presented by the beginning of the 5th semester
5–9	1 seminar	PhD research	PhD phase begins
10	1 seminar	PhD dissertation	PhD thesis must be defended by the end of the 10th semester

An individual course plan is prepared by every graduate student in cooperation with his or her academic advisor and further faculty members as appropriate. Qualified students can enter the program at various advanced stages, depending on their qualifications. For instance, the graduate committee may waive the qualifying examination for students holding an MSc degree. Students pass to the Ph.D. phase if they have received the prescribed credits, have passed the qualifier, and have the agreement of a faculty member to be their PhD advisor.

2.3 Course Requirements

In order to obtain a PhD or MSc degree, a student has to satisfy the following coursework requirements (in addition to the general Jacobs University requirements):

1. For a PhD degree: graduate courses and seminars worth at least 95 ECTS credits
2. For a Master's degree: graduate courses and seminars worth at least 95 ECTS credits
3. For both degrees: the courses Introductory Algebra (100 321) or Algebra (100 421), Real Analysis (100 313), and Introductory Complex Analysis (100 312) or Topics in Complex Analysis (100 412).

Throughout their studies, graduate students are required to take one graduate level seminar each semester. In addition, all graduate students are expected to regularly attend the mathematics colloquium.
Graduate classes are 400 level courses and above; up to five 300 level undergraduate courses may be counted towards the course credit for PhD or Master's degrees.
Courses at 300 and 400 level carry 7.5 ECTS credits; advanced graduate seminars carry 5 ETCS credits. A research proposal, including presentation, carries 25 ECTS credits. The Master's thesis carries 25 ECTS credits as well.

2.4 Qualifying Examination

Every graduate student working towards a PhD in mathematics must pass a comprehensive examination in mathematics at the beginning of the fourth semester. By the end of the third semester, the student has to request from the coordinator of the graduate program in mathematical sciences time and date for her or his examination.
The purpose of this examination is to manifest solid knowledge of advanced but core material in mathematics and/or applied and computational mathematics, to show the ability to make connections between areas of mathematics usually taught in different courses, and to demonstrate the potential for research in the mathematical sciences.
Students are free to choose between two examination formats, Mathematics or Applied and Computational Mathematics.
Both examination formats are oral, the exam takes at least 90 minutes. The material covered is described below according to format.

Mathematics

1. Algebra
2. Real Analysis
3. Complex Analysis

plus a choice of one among the following topics

4. Topology
5. Numerical Analysis
6. Partial Differential Equations
7. Mathematical Physics
8. Functional Analysis
9. Probability Theory

Applied and Computational Mathematics

1. Concepts from Algebra and Complex Analysis
2. Real Analysis
3. Numerical Analysis

plus a choice of one among the following topics

4. Scientific Computing
5. Partial Differential Equations
6. Mathematical Physics
7. Functional Analysis
8. Probability Theory

The syllabi for all topics are presented in Section 3. The examination is given by three professors appointed by the graduate committee.

2.5 Master's Option

Any graduate student may at any time request to work for a Master's degree, independently of whether or not he or she continues to work for a PhD degree.
The graduation requirements are specified by the Jacobs University's policies; the coursework requirements are described in Section 2.3. There is no separate Master's examination or qualifying examination.
The following table describes a study plan for a graduate student who has entered the program with a Bachelor's degree and wishes to conclude his or her graduate education with a Master's degree after four semesters.

Semester	Coursework	Research
1–2	3 courses and 1 seminar	
3	3 courses and 1 seminar	Preliminary work
4	1 course and 1 seminar	Master's thesis

3 Courses

The graduate courses offered in the first three semesters of the graduate program in Mathematical Sciences have two purposes. The first is to provide a solid and broad foundation of mathematical knowledge that is needed for doing research in mathematical sciences. The second purpose (at least of some of the courses) is to also provide an introduction to a specific area of current research, so as to give the students some basis for their decision on their area of specialization.

3.1 300 Level Courses

Depending on their preparation, students may decide to enroll in a certain number of 300 level courses that provide the necessary pre-requisites for many 400 level graduate courses and also cover substantial portions of the qualifying examination syllabi (see Section 4 below). For example, the courses

100312 Introductory Complex Analysis
100313 Real Analysis
100321 Introductory Algebra
100353 Manifolds and Topology
100382 Stochastic Processes
110313 Numerical Analysis

cover important aspects of a solid graduate education in Mathematical Sciences. The syllabi of 300 level courses can be found in the handbooks of the undergraduate majors Mathematics (100xxx) and Applied and Computational Mathematics (110xxx). Students should consult with their initial academic advisers on their choice of 300 level courses.

3.2 400 Level Courses

100412 – Topics in Complex Analysis

Short Name:	CompAnalysis
Type:	Lecture
Semester:	1 through 4
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents Topics in Complex Analysis builds on the material taught in the undergraduate Complex Variables course. After a quick review of the most important results and concepts, some more advanced topics are covered. Possible subjects are Riemann Surfaces, Elliptic Functions and Modular Forms, Complex Dynamics, Geometric Complex Analysis, or Several Complex Variables. Which subjects are chosen will depend on the instructor and on the students' interests. This course may also provide an introduction to a specific area of research, leading to possible PhD thesis projects.
Due to the varying content, this course can be taken multiple times for credit.

100421 – Algebra

Short Name: Algebra
Type: Lecture
Semester: 1 through 4
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Advanced topics from algebra, including groups, rings, ideals, fields, and modules, continuing the course Introductory Algebra (100 321).

100422 – Advanced Algebra

Short Name: AdvAlg
Type: Lecture
Semester: 1 through 4
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course develops more advanced topics in algebra beyond those from the Algebra course (100 421), including Galois theory, commutative algebra and its relation to algebraic geometry, as well as elements of noncommutative algebra.

100431 – Number Theory

Short Name: NumberTheory
Type: Lecture
Semester: 1 through 4
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course is mainly an introduction to algebraic number theory, but it also covers some analytic number theory, most notably the Dedekind zeta function and the analytic class number formula. Topics include algebraic number fields and their rings of integers, ideal theory in Dedekind rings, localization, p-adic numbers and fields, ideal class group and unit group, finiteness of the class number, Dirichlet unit theorem, Dedekind zeta function, analytic class number formula, perhaps Dirichlet L-series and a proof of Dirichlet's theorem on primes in arithmetic progressions, Artin reciprocity with the main results (no proofs) of class field theory.

100432 – Algebraic Geometry

Short Name: AlgGeometry
Type: Lecture
Semester: 1 through 4
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Algebraic geometry is the study of geometry using algebraic tools: the geometric objects are the common roots of a set of polynomials in several variables. Many geometric properties can be studied in terms of algebraic properties of these polynomials, using the powerful machinery of algebra to study geometry.
Basic concepts from 100 421 (Algebra) and 100 321 (Introductory Algebra) are used in this course. Among the studied subjects are affine and projetive varieties, schemes, curves, and cohomology.

100442 – Algebraic Topology

Short Name: AlgebrTopology
Type: Lecture
Semester: 1 through 4
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course is mostly concerned with the comprehensive treatment of the fundamental ideas of singular homology/cohomology theory and duality. The knowledge of fundamental concepts of algebra as well as of general topology is assumed (at a level of Introductory Topology and Introductory Algebra).
The first part studies the definition of homology and the properties that lead to the axiomatic characterization of homology theory. Then further algebraic concepts such as cohomology and the multiplicative structure in cohomology are introduced. In the last section the duality between homology and cohomology of manifolds is studied and few basic elements of obstruction theory are discussed.
The graduate algebraic topology course gives a solid introduction to fundamental ideas and results that are used nowadays in most areas of pure mathematics and theoretical physics.

100451 – Differential Geometry

Short Name:	DiffGeom
Type:	Lecture
Semester:	1 through 4
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents Differential geometry is the study of differentiable manifolds. Assuming basic concepts from 100 311 (Integration and Manifolds) and 100 351 (Introductory Geometry), such as manifolds, differential forms, and Stokes' theorem, the focus in this course is on Riemannian geometry: the study of curved spaces which is at the heart of much current mathematics as well as mathematical physics (for example, General Relativity).

100452 – Lie Groups and Lie Algebras

Short Name:	LieGroups
Type:	Lecture
Semester:	1 through 4
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents A Lie group is a group with a differentiable structure, the tangent space at the identity element of a Lie group is its Lie algebra. Lie groups and Lie algebras are indispensable in many areas of mathematics and physics. As a mathematical subject on its own, Lie theory has led to many beautiful results, such as the famous classification of semisimple Lie algebras. In physics, Lie groups and their representations are essential to the theory of elementary particles and its current developments. Due to the close correspondence of physical phenomena and abstract mathematical structures, the theory of Lie groups has become a showcase of mathematical physics.
The course presents fundamental concepts, methods and results of Lie theory and representation theory. It covers the relation between Lie groups and Lie algebras, structure theory of Lie algebras, classification of semisimple Lie algebras, finite-dimensional representations of Lie algebras, and tensor representations and their irreducible decompositions.
A solid background in multivariable real analysis and linear algebra is presumed. Familiarity with some basic algebra and group theory will also be helpful. No prior knowledge of differential geometry is necessary.

100453 – Modern Geometry

Short Name: ModernGeometry
Type: Lecture
Semester: 1 through 4
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents The course serves as an introduction, at the advanced level, to the basic concepts of modern geometry. It supplements the following sequence of courses: Introductory Geometry, Manifolds and Topology, Differential Geometry, and Algebraic Geometry. The new course fits as the third in this sequence of courses, right after the 300-level courses, Introductory Geometry, Manifolds and Topology, and before the 400-level courses, Differential Geometry and Algebraic Geometry.
The following concepts, known from the 300-level courses, should be briefly reviewed: concept of a manifold, the simplest examples of manifolds, and the concept of homotopy.
The core of the course will consist of explaining material related to the following topics: Lie groups, homogeneous spaces, symmetric spaces, fiber bundles, for example, vector bundles, Morse theory, differential topology of mappings and submanifolds.
This material will provide a solid background for the 400-level courses, Differential Geometry and Algebraic Topology.

100461 – Dynamical Systems

Short Name: DynSystems
Type: Lecture
Semester: 1 through 4
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Based on the undergraduate ODE/Dynamical Systems course, this course goes more deeply into the theory of discrete and continuous dynamical systems. Possible topics include bifurcation theory, stable and unstable manifolds, KAM theory, or the shadowing lemma. This course may also provide an introduction to a specific area of research, leading to possible PhD thesis projects.

100471 – Functional Analysis

Short Name:	FunctAnalysis
Type:	Lecture
Semester:	1 through 4
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents This course assumes basic knowledge of measure and integration theory, and of classical Banach and Hilbert spaces of measurable functions. Functional Analysis focuses on the description, analysis, and representation of linear functionals and operators defined on general topological vector spaces, most prominently on abstract Banach and Hilbert spaces. Even though abstract in nature, the tools of Functional Analysis play a central role in applied mathematics, e.g., in partial differential equations. To illustrate this strength of Functional Analysis is one of the goals of this course.

110411 – Topics in Applied Analysis

Short Name:	ApplAnalysis
Type:	Lecture
Semester:	1 through 4
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents The course Topics in Applied Analysis introduces to a variety of fundamental analytic tools and methods used in the theory, modeling, and numerical simulation of phenomena in the natural sciences. The course is offered with different contents in different years, the choice will depend on the instructor. Examples of areas currently covered are applied harmonic analysis and operator theory, perturbation theory and asymptotic analysis, approximation theory, and others. Students specializing in applied mathematics or applied sciences may participate in this course more than once.

3.3 Reading Courses in Mathematics

Specialized topics, often related to faculty research areas, are taught in the form of reading courses. Offers depend on student and faculty interests. As a rule, reading courses carry 5 credits. Usage of these credits towards a graduate degree requires approval by the graduate education committee.

3.4 Graduate Seminars

100591/100592 – Mathematics Colloquium

Short Name:	MathColloquium
Type:	Seminar
Semester:	All
Credit Points:	None
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents The weekly mathematics colloquium features talks by international scientists for the entire mathematical community, broadening horizons and encouraging formal or informal interactions.

100491/100492 – Graduate Research Seminar

Short Name:	GradResearchSem
Type:	Seminar
Semester:	1
Credit Points:	5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents This course is intended for beginning graduate students to help them identify interesting areas of research and possible thesis subjects and advisors. It consists of lectures mainly by professors, but also by other faculty, about current areas of research in mathemaical sciences, with particular emphasis on research areas of Jacobs faculty. Students get involved in discussions of all the areas of research; during the course of the semester, they choose at least three topics which they investigate further and which they elaborate into a research report. At the end of the semester, every student presents at least one of these reports. Participation is also open for advanced undergraduates looking for topics for their undergraduate theses, the results of which are presented as well.
Graduate research seminar participants receive a grade on the scale 1.0 through 5.0.

Advanced Seminars

In addition, there are regular research seminars run by the faculty of the graduate program on advanced subjects and/or on topics of current research interests. Such seminars carry 5 ECTS each; as a rule, participants receive Pass/Fail grades.

4 Qualifying Examination Syllabi

4.1 Algebra

1. **Groups:** monoids, groups, cyclic groups, normal subgroups, operation of a group on a set, Sylow subgroups, direct sums and free abelian groups, finitely generated abelian groups, nilpotent and solvable groups, Jordan- Hölder's theorem.
2. **Rings:** rings and homomorphisms, ideals, commutative rings, localization, principal rings, PIDs, Euclidean rings, UFD's.
3. **Polynomials:** Euclidean algorithm, unique factorization in several variables, Hilbert's Nullstellensatz, criteria for irreducibility, the derivative and multiple roots, symmetric polynomials.
4. **Modules:** homomorphisms, exact sequences, direct sums of modules, free modules, projective and injective modules, modules over PIDs, tensor product of modules.
5. **Noetherian rings and modules:** basic criteria, Hilbert's basis theorem, power series.
6. **Fields and Galois theory:** finite and algebraic extensions, algebraic closure, splitting fields and normal extensions, separable and Galois extensions, main theorem of Galois theory, finite fields, primitive elements, purely inseparable extensions (at least know one example), the norm and trace, cyclic extensions, solvable and radical extensions. (Including the standard applications: regular n-gons, trisecting an angle, ...).

Main reference
Thomas W. Hungerford , Algebra, Springer.

Alternative and complementary reading
Serge Lang, Algebra, Springer.

4.2 Real Analysis

1. **Measures**: algebras, sigma algebras, measures, outer measures, premeasures, continuation and completion, Borel and Lebesgue measures on $\mathbb{R}^d$.
2. **Integration**: measurable functions, integrability of real and complex valued functions, limit theorems, modes of convergence, product measures and Fubini-Tonelli theorem, the Lebesgue integral on $\mathbb{R}^d$.
3. **Signed measures**: Hahn and Jordan decomposition theorems, absolute continuity, Radon Nikodym theorem, bounded variation.
4. L^p **spaces:** basic inequalities (Hoelder, Minkowski, Jensen, Chebyshev), bounded linear functionals on L^p, Hardy-Littlewood maximal function.
5. **Elements of functional analysis**: Hilbert spaces, complete orthonormal systems, Fourier series on $\mathbb{T}^d$, Fourier transform on $\mathbb{R}^d$.
6. **Elements of distribution theory**: $D(\mathbb{R}^d)$ and $D'(\mathbb{R}^d)$, Schwarz class and tempered distributions, basic operations on distributions.

Main reference
Gerald Folland, Real Analysis: Modern Techniques and Their Applications, 2^{nd} Edition, John Wiley & Sons.

Alternative and complementary reading
W. Rudin, Real and Complex Analysis, 3^{rd} Edition, Mc Graw - Hill.
E. H. Lieb, M. Loss, Analysis, 2^{nd} edition, AMS 2001 (Chapters 1-6).
H.L. Royden, Real Analysis, 3^{rd} Edition, Macmillan.

4.3 Complex Analysis

The material covered in *Complex Analysis, Lars Ahlfors,* 3^{rd} *Edition, Mc Graw - Hill*

4.4 Concepts from Algebra and Complex Analysis

This qualifier covers the fundamental concepts from the Algebra and Complex Analysis qualifiers.

Concepts from Algebra

1. **Groups:** monoids, groups, cyclic groups, normal subgroups, operation of a group on a set, direct sums and free abelian groups, finitely generated abelian groups.
2. **Rings:** rings and homomorphisms, ideals, commutative rings, principal rings, PIDs, Euclidean rings, UFDs.
3. **Polynomials:** Euclidean algorithm, unique factorization in several variables.
4. **Fields and Galois theory:** finite and algebraic extensions, algebraic closure, splitting fields and normal extensions, separable and Galois extensions, main theorem of Galois theory, finite fields.

Concepts from Complex Analysis
The material covered is Chapters 1 to 4, and Sections 1 and 2 of Chapter 5 in *Complex Analysis, Lars Ahlfors,* 3^{rd} *Edition, Mc Graw - Hill.*

Main reference
T.W. Hungerford , Algebra, Springer.
L. Ahlfors, Complex Analysis, 3^{rd} Edition, Mc Graw - Hill

Alternative and complementary reading
S.Lang, Algebra, Springer.
J.B. Conway, Functions of one complex variable, 2^{nd} Edition, Springer.

4.5 Topology

The material covered in *Topology, Klaus Jänich, Undergraduate Texts in Mathematics, Springer*

4.6 Numerical Analysis

1. **Basics of error, computational work, and condition/stability analysis.**
2. **Numerical linear algebra:** direct (Gauss/LU/QR) and iterative solvers (GS, ILU, PCG, GMRES), linear least-squares problems, eigenvalue solvers.

3. **Numerical methods of analysis:** integration, interpolation and approximation, FFT.
4. **Nonlinear equations and optimization:** Newton-type methods, nonlinear least-squares methods, global search methods, unconstrained and constrained smooth optimization.
5. **Initial value problems for ODEs:** implicit and explicit one- and multi-step methods, extrapolation methods, convergence and stability concepts, stiff problems.
6. **PDE discretization:** finite difference and finite element method for elliptic and parabolic PDE, solution of sparse linear systems.

Main reference
A. Quarteroni, R. Sacco, F. Saleri, Numerical Mathematics, Springer.

Alternative and complementary reading
G. Golub, C. van Loan, Matrix Computations, 3^{rd} edition, Johns Hopkins.
E. Hairer, S.P. Norsett, G. Wanner, Solving Ordinary Differential Equations I,II, Springer.
S. Larsson, V. Thomee, Partial Differential Equations with Numerical Methods, Springer.
J. Nocedal, S.J. Wright, Numerical Optimization, 2^{nd} Edition, Springer.

4.7 Partial Differential Equations

1. **Linear model equations:** Transport, Laplace, Heat, Wave Equations (classical solution techniques, representation formulas, energy methods).
2. **First-order Nonlinear Equations:** method of characteristics, introduction to Hamilton-Jacobi equations and conservation laws.
3. **Sobolev spaces and elliptic boundary value problems:** existence and regularity of weak solutions, weak and strong maximum principles.
4. **Linear evolution equations:** weak solutions of parabolic and hyperbolic equations, semi-group techniques.
5. **Variational and Nonvariational Methods for Nonlinear PDE:** existence and regularity of minimizers and critical points of energy functionals associated with PDE.

Main references
L. C. Evans, Partial differential equations, Grad. Studies in Mathematics v.19, AMS. Part I + II + Selected Topics from Part III.

Alternative and complementary reading
J.Jost, Partial differential equations, Grad. Texts in Mathematics, Springer, 2002.

4.8 Mathematical Physics

Syllabus offered by Professors Schupp and Huckleberry upon request.

4.9 Functional Analysis

1. **Topological vector spaces and completeness:** Baire category theorem, open mapping theorem, closed graph theorem.

2. **Convexity, weak topologies, duality in Banach spaces and compact operators:** Hahn-Banach theorem, Banach-Alaoglu theorem, Krein-Milman theorem.
3. **Distributions, Fourier analysis and applications to differential equations:** Haar measure on compact groups, Fourier analysis on Groups, Fourier transforms, Fourier series, distributions and tempered distributions.
4. **Banach algebras and spectral theory:** Gelfand-Mazur theorem, commutative Banach algebras, Gelfand transforms, bounded operators on a Hilbert space, spectral theorem for normal and bounded operators.

Main references
W. Rudin, Functional Analysis, 2^{nd} Edition, McGraw-Hill.
J.B. Conway, A Course in Functional Analysis, 2^{nd} Edition, Springer.

Alternative and complementary reading
G. Folland, Real Analysis, Wiley.
W. Rudin, Fourier Analysis on Groups, Wiley, pages 1 to 13.

4.10 Probability Theory

1. **General probability spaces:** Discrete and geometric probabilities.
2. **Random variables:** Joint distribution function, density (if exists), quantiles. Examples, discrete and continuous (uniform, exponential, normal).
3. **One-dimensional transformations of distributions.**
4. **Expectation of a random variable (and its function):** Variance, moments.
5. **Joint distributions and independence:** marginal distributions, joint density.
6. **Infinite sequences of random variables:** Borel-Cantelli lemma. Modes of convergence. Convergence of expectations. Weak and strong laws of large numbers, central limit theorem.
7. **Joint distributions:** conditioning, correlation, and transformations. Conditional distributions, conditional expectation. Total probability formula (continuous case). Regression and correlation. Multidimensional transformations of distributions. Distribution and density of sum, product and quotient of one-dimensional random variables.
8. **Countable Markov chains, Random walks.**
9. **Classification of finite Markov chains.**
10. **Brownian motion:** Construction and basic properties.

Main references
L.B. Koralov, Y.G. Sinai, Theory of Probability and Random Processes, Springer.

Alternative and complementary reading
A.N. Shiryaev, Probability, 2^{nd} Edition, Springer.

4.11 Scientific Computing

1. **Large scale scientific computing:** computational and analytical aspects of FEM, FDM, FVM, multigrid method, domain decomposition method, error estimators, adaptivity, solution of sparse systems.
2. **High performance computing:** principles of parallel and distributed computing, computation on graphics hardware, data structures, complexity and runtime efficiency of standard algorithms (FFT, dense and sparse solvers for numerical linear algebra)
3. **Application area, select one from:** a) Modeling with PDEs, b) Signal and Image Processing, c) Scientific Visualization.

Main references

A. Quarteroni, R. Sacco, F. Saleri, Numerical Mathematics, Sringer.
S. Larsson, V. Thomee, Partial Differential Equations with Numerical Methods, Springer 2003.
A. M. Bruaset, A. Tveito. Numerical solution of partial differential equations on parallel computers, Springer 2006.

Alternative and complementary reading

S. Salsa. Partial Differential equations in action: from modeling to theory. Springer.
G. Aubert, P. Kornprobst, Mathematical Problems in Image Processing – Partial Differential Equations and the Calculus of Variations, Springer.

5 Faculty Research Areas

Complex geometry
Prof. Alan Huckleberry

Group theory and geometry
Dr. Mallahi Karai

Affine algebraic geometry, commutative algebra, cryptography
Dr. Stefan Maubach

Quasiconformal uniformization and dynamics
Prof. Daniel Meyer

Partial differential equations and fluid dynamics
Prof. Marcel Oliver

Approximation theory, numerical analysis, multiscale methods
Prof. Peter Oswald.

Algebra, geometry, Lie theory, and representation theory
Prof. Ivan Penkov

Modeling of bio-medical processes with partial differential equations
Prof. Tobias Preusser

Applied harmonic and functional analysis, wavelets, Gabor theory, signal processing
Prof. Götz E. Pfander

Dynamical systems, conformal and fractal geometry
Prof. Dierk Schleicher

Mathematical and theoretical physics
Prof. Peter Schupp

Coxeter groups, hyperbolic geometry, cluster algebras
Dr. Pavel Tumarkin.

Mathematical physics, Riemannian geometry, wavelet theory
Prof. Raymond O. Wells

5.1 Detailed description of research areas

Alan Huckleberry *Complex geometry*
Originally “Funktionentheorie” meant the study of functions of one complex variable which satisfy the Cauchy-Riemann differential equations. Already in the mid-19th century Riemann introduced the appropriate global geometric setting for this study, in particular Riemann surfaces which are in today’s jargon 1-dimensional complex manifolds. In the early part of the 20th century it was realized that a higher-dimensional version of this theory would have wide-ranging applications in mathematics and various natural sciences. The foundations of the geometric side of this multi-dimenensional theory, “Complex Geometry”, were systematically developed in the mid- to late-20th century. The interplay with algebraic geometry, symplectic geometry and its Hamiltonian viewpoint and differential geometry, e.g., on manifolds with special metrics, became a way of life. In recent years the subject has become focused on special settings which arise in, e.g., mathematical physics. These often involve considerations of symmetry, the study of classical motion and the analysis of associated group representations. Symmetry in complex geometry and related aspects of classical and quantum mechanics are dominating factors in my present research interests.

Mallahi Karai *Group theory and geometry*
My mathematical interests cluster around the interactions between the theory of infinite groups and several other areas in mathematics. If G is an infinite finitely generated group, G can be furnished with the word metric which endows G with a large-scale geometry. Following Gromov’s philosophy, we are interested in connections between the algebraic properties of the group G that are reflected in its large-scale geometry.
In a different direction, Kazhdan introduced property T to study some algebraic properties of the lattices in (higher rank) semi-simple Lie groups. Defined using the representation theory of G, this property has turned out to have unsuspecting applications in combinatorics. The first explicit construction of the expanders takes advantage of the fact that arithmetic lattices enjoy this property. I am interested in studying property T for various classes of groups, including groups related to the automorphism group of the free group.

Stefan Maubach *Affine algebraic geometry, commutative algebra, cryptography*
Affine Algebraic Geometry (short AAG) concerns itself with affine spaces, which are, in essence, commutative rings, mostly polynomial rings over a field. Affine spaces are the building blocks of “generic” algebraic geometry. This still growing topic has become a distinct research field since the late 1980s, and received its own AMS classification in 2000. A space like $\mathbb{A}^n$ is the simplest example of a so-called affine variety, and a very important one. In contrast to projective geometry and scheme-theoretic geometry, affine algebraic geometry is closer to algebra. There is a one-to-one correspondence between affine spaces (varieties) and quotients of polynomial rings. Hence, in this field, it is very beneficial to switch back and forth between a geometric and algebraic viewpoint, making it a truly vibrant, interdisciplinary field.
Let me classify my current research in some sub-topics:
Finite fields and cryptography: I try to understand the automorphism group of $\mathbb{F}_p[X_1, \ldots, X_n]$. Recently, I’ve been studying quotients of this automorphism group. This topic also includes trying to understand when an endomorphism is an automorphism. Furthermore, I am developing an application to symmetric key cryptography, more in particular a system to create

session-keys.
LF automorphisms: When a map F satisfies F^n depends linearly on $F^{n-1}, F^{n-2}, \ldots, F, I$ then F is a locally finite map (short LF map). The set of LF automorphisms turns out to be an interesting class, generating many and perhaps all automorphisms.
Modern invariants: I study the Makar-Limanov and Derksen invariants of rings, the former being the intersection of all kernels of locally nilpotent derivations on that ring. One of the motivations for this is to find counterexamples to the Cancelation Problem: If V is an affine variety such that $V \times \mathbb{C} \cong \mathbb{C}^n$, is $V \cong \mathbb{C}^{n-1}$? Related, I am interested in studying the set of subrings of a ring, and the action of the automorphisms of the ring induced on this set of subrings.

Daniel Meyer *Quasiconformal uniformization and dynamics*
My research lies in the area of Geometric Function Theory. This means I am particularly interested in the interaction between the analytic description and the geometry of an object. Particularly I am interested in quasiconformal maps. These are generalizations of conformal, i.e., angle-preserving maps. They play a fundamental role in Complex Analysis, as well as in Teichmller Theory, Complex Dynamics, and Kleinian Groups. They also appear naturally in Geometric Group Theory.
An important problem is that of quasiconformal uniformization. This asks when some metric space is quasiconformally equivalent to some standard space. Thus one looks for analogs of the Riemann mapping theorem.
I have shown that a large class of metric spheres are quasispheres, i.e., quasiconformal images of the unit sphere. This is the largest as well as the best understood class. The question when a given metric sphere is a quasisphere is particularly relevant to Cannon's conjecture, a well-known open problem from Geometric Group Theory. It asks whether a group that acts topologically as a Kleinian group is in fact a Kleinian group.
There is a close connection between the dynamics of Kleinian groups and rational maps, known as Sullivan's dictionary. A famous theorem by Thurston gives an answer to the question corresponding to Cannon's conjecture in the rational map case. Namely it gives (necessary and sufficient) criteria when a map that acts topologically as a rational map acts geometrically as a rational map. In joint work with Mario Bonk I have started a systematic study of rational maps from a viewpoint that closely mirrors the one from geometric group theory. One of the successes of this theory has been that I was able to show that every map (from a suitable class of rational maps) has an iterate that arises as a mating. This means one can decompose such a map geometrically in two polynomials. The corresponding problem from Kleinian groups is called the virtual fibering conjecture, which is still wide open.

Marcel Oliver *Partial differential equations, fluid dynamics*
Structure Preserving Numerical Algorithms: When solving complex problems numerically, it is often not possible to resolve all features of the solution to a high accuracy. It is therefore necessary to design algorithms which preserve salient features to high accuracy, while being less accurate on others.
This can be done in two ways. First, by designing numerical algorithms which possess discrete analogs of the structure, for example the conservation laws, of the continuum model. The mathematical task is then to prove that this leads to improved accuracy in the simulation of those continuum features. Alternatively, one can try to first reduce the continuum model to

a simplified, but still fully space-time continuous equation, before using a standard numerical method for the simulation.
Both approaches are currently being explored, particularly in the context of variational integrators for nonlinear wave equations, Lagrangian methods in fluid dynamics, and variational approaches to balance models in geophysical fluid dynamics.
Transverse phase-instabilities for planar traveling fronts: Traveling planar fronts of a two-component autocatalytic reaction-diffusion system become unstable with respect to transverse perturbations as the ratio of the diffusivities of the two components is increased beyond a certain threshold. When increased even further, it is known through full numerical simulation that the wrinkling of the front develops patterns on two distinct spatial scales.
We are currently studying, by direct numerical simulation as well as using numerical Evans function techniques, if this phenomenon can be attributed to a linear instability with two distinct linearly maximally unstable wave numbers of comparable growth exponents, or else must be explained with a fully nonlinear theory.

Peter Oswald *Approximation theory, numerical analysis, multiscale methods*
My research is driven by both scientific curiosity and applications in areas such as geometric modeling and computer graphics, data analysis, processing and compression, communication theory, adaptive and optimal-complexity algorithms for large-scale simulations in engineering and natural sciences, and others.
Current projects are:
Subdivision has originally been introduced for the fast evaluation of spline functions, and later turned into a tool for creating hierarchical representations of discrete curves and triangulated surfaces in CAGD and computer graphics software. The quality of a subdivision curve or surface depends on the ingredients (mesh refinement and local averaging) of a subdivision scheme in a nonlinear way but can be studied using so-called vector refinement equations. We are currently interested in the design of good subdivision methods with non-standard and irregular refinement rules, the analysis of mixed schemes, and the stability and smoothness for nonlinear subdivision methods that have recently surfaced in shape-preserving approximation, data denoising, image and geometry compression.
Multi-grid algorithms, multilevel finite element solvers, and other wavelet-type techniques have become standard in large-scale simulation efforts based on systems of partial differential equations. We are interested in the understanding of their potential and performance limits for PDE with variable coefficients, for hp-type discretizations, on unstructured mesh sequences, etc.
Research in Nonlinear Approximation Theory currently aims at quantifying the performance gains from and finding improvements for approximation schemes with built-in adaptivity capabilities. Traditional mesh adaptivity in PDE solvers is studied via basis function selection, and greedy algorithms are analyzed in various settings. Tensor-product techniques for use in high-dimensional problems, in particular in connection with learning theory, data analysis and compression, will be pursued.

Ivan Penkov *Algebra, geometry, Lie theory, representation theory*
Lie groups are continuous groups, i.e. manifolds with group structure, and a Lie algebra is the tangent space of a Lie group at unity. The structure theory of Lie groups and Lie algebras, developed by W. Killing, E. Cartan and H. Weyl in the first half of the 20th century, belongs to the jewels of modern mathematics. This theory is also a standard tool of today’s mathematical

physics. A more recent fundamental achievement of Lie theory is the complete description of a class of representations of Lie groups and Lie algebras of fundamental importance. There are the so called Harish Chandra modules, and their classification was completed in the early 1980's by a tour de force using sophisticated algebraic and geometric techniques.
Many deep problems in the structure theory of representations of Lie algebras and Lie groups are still open. One such problem is the classification of all simple modules M over simple matrix Lie algebras, satisfying the condition that M decomposes as a direct sum of finite dimensional isotypic components over a suitable subalgebra. In the late 1990's my collaborators and I gave these representations the name generalized Harish Chandra modules and initiated a program to study them. The main achievement of this program has been the construction of large classes of new generalized Harish Chandra modules, as well as the description of arbitrary reductive subalgebras over which generalized Harish Chandra modules can have finite dimensional isotypic components. Currently I am actively pursuing this program.
Another program, in which I am actively working, is the structure theory of a class of infinite dimensional Lie algebras and their representations. These are the classical locally finite Lie algebras. The goal of my collaborators and myself is to develop a structure theory which would have the same detail as the classical structure theory of finite dimensional Lie algebras. Our recent successes have been a complete description of Cartan and Borel subalgebras (the latter is still in progress), construction of the theory of weight modules, an infinite dimensional version of Borel-Weit-Bott theory, and a construction of the most general highest weight modules. These results are based on innovative infinite dimensional techniques as generalized flags and their corresponding homogeneous ind-spaces.
Finally, I am pursuing also a program in infinite dimensional algebraic geometry. More specifically, I am studying vector bundles on homogeneous ind-spaces. A success if this program is the result that, on certain ind-grassmannians all vector bundles of finite dimension are homogeneous.
My entire scientific program offers wide opportunities for undergraduate and graduate research topics.

Götz Pfander *Applied harmonic analysis, wavelets, Gabor theory, signal processing, communications engineering*
Time-frequency analysis of operators: The objects most prominently studied using time-frequency analysis are functions defined on Euclidean spaces and their decompositions. Many of the fundamental questions on Gabor systems, i.e., on systems generated from a finite number of prototype functions by time and frequency shifts, can be rephrased to address operators associated to these systems. E.g., a function system is a frame if the frame analysis operator is bounded and stable.
We are applying results from Gabor frame theory to the study of a variety of different operator classes such as general Hilbert-Schmidt operators or underspread operators and their decompositions in time-frequency shift operators (and vice versa). Realizations of uncertainty principles are in particular interesting to us.
Gabor and wavelet systems: Orthogonal (complete) wavelet systems and their applications have become a very popular field of research in applied mathematics during the last two decades. The flexibility given by overcomplete [resp. undercomplete] Gabor or wavelet frames [resp. Riesz systems] is nowadays proving to be more and more helpful in many parts of signal synthesis and analysis. In fact the redundancy present in overcomplete systems can be used to reduce

effects of certain disturbances in a communications system.
We are studying coherent function families such as wavelet and Gabor systems with respect to their applicability for information transmission in certain linear time invariant and in slowly time varying channels as present in mobile communication systems. We are interested in both, results concerning the structure of the coherent function systems and in the design of appropriate wavelets and Gabor window functions.

Tobias Preusser *Modeling of bio-medical processes with partial differential equations*
My research interests lie on the one hand in the field of modeling, simulation and optimization with partial differential equations and the efficient numerical implementation using multiscale approaches, multigrid methods and parallel computing. On the other hand my research is driven by concrete and complex application scenarios in medicine. The goal is to enhance radiological image data of patients by individual, robust and clinically applicable models, simulations and optimizations in order to allow for a better quality in diagnosis and treatment of diseases. This field of mathematical modeling, simulation and optimization in medicine is a young but rapidly growing research discipline, which is accelerated by the recent advances in digital medical imaging technologies.
At the Fraunhofer Institute for Medical Image Computing MEVIS I am leading the group on ”Modeling and Simulation”. Fraunhofer MEVIS is a worldwide renowned research institute, which aims at developing software prototypes for patient specific diagnosis and treatment of diseases together with a worldwide clinical network. My group has a large expertise in the modeling, simulation and optimization of tumor treatment by thermal ablation in the human liver. We are also working on the modeling and simulation of the physiology (i.e. function), of the liver and of the flow of blood in major vessels. The Modeling and Simulation group at Fraunhofer MEVIS is an inspiring environment for a wealth of applied undergraduate and graduate research projects.

Dierk Schleicher *Dynamical systems, conformal and fractal geometry, complex variables, hyperbolic geometry*
Dynamical systems: My main research focus are dynamical systems generated by the iteration of (mainly complex differentiable) functions. A prime example is the famous and classical Newton method for finding zeroes of differentiable functions f. Even for the fundamental case when f is a polynomial, there is no satisfactory theory yet of the global behavior of the Newton method as a dynamical system, but there has been recent progress on determining a near-optimal collection of good starting points, and on proving the first rigid bound on the iteration time needed to reach prescribed precision.
A second direction concerns the iteration of transcendental entire functions, generalizing the now-classical theory for polynomials to maps of infinite degree. The prototypical case are exponential maps, where a complete construction and classification of dynamic rays and attracting dynamics has now been achieved.
Symbolic dynamics: Symbolic dynamics has always been a powerful tool for complex dynamics; a lot of work has been done in this direction by many people, mainly for the case of quadratic polynomials. The monograph Symbolic dynamics of quadratic polynomials, written jointly with Henk Bruin, has been finished as a preprint of the Institute Mittag-Leffler. It is an attempt to bring the various approaches of many people together and contains complete solutions of many of the remaining problems in the area.

Peter Schupp *Mathematical / theoretical physics*
There are two main directions of specialization for graduate students interested in Mathematical Physics: Classical Mathematical Physics (rigorous approaches to problems from various fields of physics): Core courses are Real Analysis and Quantum Field Theory. Modern Mathematical Physics (development of models and theories of fundamental physics and study of their implications): Core courses are Quantum Field Theory and Differential Geometry. The courses are supplemented by a choice of other courses from the mathematical science graduate program as well as by special subject courses of the other programs at IUB depending on the field of interest. The Mathematics Colloquium and the Theory Seminar provide contact to current research. My research interests lie in theoretical particle physics, quantum field theory, and mathematical physics. I am currently working on non-commutative quantum field theory, deformation quantization, and non-commutative geometry - in particular as a description of space-time geometry at ultra-short distances including gravity. I am always interested in challenging problems in mathematical physics and have, e.g., worked on quantum spin systems, strongly correlated electrons, and coherent states. In the context of string theory I am particularly interested in matrix theory and D-branes/M-branes (DBI action, Gerbes).

Pavel Tumarkin *Coxeter groups, hyperbolic geometry, cluster algebras*
Coxeter groups are essentially groups generated by reflections. The most known examples of Coxeter groups are groups of permutations and symmetry groups of Platonic solids. Coxeter groups appear in many branches of mathematics and physics. One of the problems arising, in particular, in physics, is to investigate subgroups of Coxeter groups generated by reflections. Classification of subgroups is still an open problem, as well as many interesting partial cases, for example, classification of subgroups of finite index.
A very interesting application of Coxeter groups can be found in recently developed theory of cluster algebras. The theory of cluster algebras itself has already found various applications in many domains, such as combinatorics, representation theory, Teichmueller theory, theoretical physics, and many others. The problems of combinatorial nature arising in the theory show surprising relations with the theory of Kac-Moody algebras and Coxeter groups. One of the goals of my research is better understanding of the interplay between these theories and combinatorics of cluster algebras.