JACOBS UNIVERSITY

Mathematical Sciences

Integrated PhD Program

Mathematical Sciences

Graduate Progam Handbook

Academic Year 2014–15

Contents

As of September 1, 2014, the *School of Engineering and Science* and the *School of Humanities and Social Sciences* have been replaced by the Focus Areas *Health*, *Mobility*, and *Diversity*. Handbooks and policies might still refer to the old structure of Schools.

If this is the case, references to the School of Engineering and Science include courses offered within the following disciplines:

- Electrical Engineering and Computer Science
- Life Sciences
- Logistics
- Mathematical Sciences
- Natural and Environmental Sciences

References to the School of Humanities and Social Sciences include courses offered within the following disciplines:

- Economics and Management
- History
- Humanities
- Law
- Psychology
- Social Sciences
- Statistics and Methods

1 Concept

1.1 General Description

The graduate program in Mathematical Sciences is part of the graduate school associated to the ”Mathematics, Modeling and Computing (MAMOC)” research center.

The Mathematical Sciences Graduate Program at Jacobs University offers the opportunity for graduate study in pure, applied and computational mathematics, as well as in mathematical physics. The program leads to a *doctorate degree (PhD)*; a *Master's degree (MSc)* may be obtained as well.

This is an “integrated PhD program” which accepts students holding a Bachelor degree, as well as more advanced students. An early beginning has the advantage that students can spend their first semesters in the program exploring research areas and meeting possible advisors before having to finalize their choice, thus making better informed decisions. More advanced students are admitted at a level compatible with their previous education.

The initial part of the program involves a broad education in mathematical science, followed by a choice of advanced courses, seminars, and research activities leading to a dissertation.

Graduate students at Jacobs University are viewed as professionals. From early on, they are integrated into the faculty's international research collaborations, they participate at international research conferences or in thematic research programs—a head start into a successful career in academia or industry.

1.2 Program Overview and Duration

Doctor of Philosophy (PhD) Students entering the graduate program with a Bachelor degree are required to complete successfully up to three semesters of coursework and a qualifying exam before progressing to the PhD dissertation. The program generally takes up to five years after the BSc degree. A separate MSc thesis is not required for students working towards a PhD degree, but students have the option to earn a separate Master's degree en route.

Students holding a Master's degree (or equivalent) typically need no more than three years until completion of their PhD degree.

Master of Science (MSc) The MSc degree requires up to three semesters of full-time coursework and one semester to produce a Master's thesis.

1.3 Interactions

Members of the Graduate Program in Mathematical Sciences interact with many faculty members and programs within the School of Engineering and Science, within Jacobs University at large, and with researchers worldwide. In particular, our weekly mathematics colloquium brings in leading mathematicians from Europe and overseas in all areas of mathematical sciences, in addition to the regular research contacts of our faculty members.

Moreover, graduate students with interests in applied, numerical, or computational mathematics are supported by Jacobs University's Computational Laboratory for Analysis, Modeling, and Visualization (CLAMV). CLAMV is equipped with advanced graphics workstations, a Linux cluster, a Sun Fire compute server, and has access to the Northern German supercomputing network. Jacobs University offers many opportunities for interaction with researchers in other fields—including geophysics, astrophysics, computer science, physics, psychology, neurosciences, and social sciences—whose work involves mathematical modeling and computation.

Graduate students with interests in Mathematical Physics can benefit from the course offerings, seminars and research activities of the Astroparticle Physics Graduate Program at Jacobs University. Traditionally there has been a strong cross-fertilization between mathematics and physics. Mathematics provides the language and forms the foundation of modern physics. Physics has inspired many important developments in mathematics. More than ever this is true today. Graduate students who want to do research in modern mathematical or theoretical physics need a strong mathematical background as it is provided in our graduate program.

1.4 Career Options

The graduate program in mathematical sciences at Jacobs University is designed to equip students with the necessary tools and scientific maturity to embark on a research career in academia or industry. Due to the central role of mathematics in science, there is a never ceasing demand for mathematicians in academia worldwide. Universities and colleges offer tenure-track and tenured positions to PhDs; certain positions are more focused on research and others more on teaching. Graduates in mathematical sciences are well sought after by non-academic employers. Consequently, mathematicians enjoy a large choice of well-regarded jobs outside of the university world, for example in research and development, finance, banking, and management.

2 Structure of the Program

Graduate education at Jacobs University is governed by the appropriate policies. Additional program specific rules are described below.

2.1 Initial Academic Advisor

Every incoming graduate student is assigned an initial academic advisor prior to coming to Jacobs University. The initial advisor guides the graduate students through the program, monitors his or her progress, and helps him or her select a PhD advisor.

2.2 Study Plan: Integrated PhD Program

The following study plan is the default variant for students entering with a BSc degree. Faster progress is always possible.

Semester	Coursework	Research	Additional Information
1–2	3 courses, 1 seminar		
3	3 courses, 1 seminar	Preliminary work	Qualifying exam must be completed at the beginning of the 4th semester
4	1 course, 1 seminar	PhD research proposal	PhD proposal must be presented by the beginning of the 5th semester
5–9	1 seminar	PhD research	PhD phase begins
10	1 seminar	PhD dissertation	PhD thesis must be defended by the end of the 10th semester

An individual course plan is prepared by every graduate student in cooperation with his or her academic advisor and further faculty members as appropriate. Qualified students can enter the program at various advanced stages, depending on their qualifications. For instance, the graduate committee may waive the qualifying examination for students holding an MSc degree.

Students pass to the Ph.D. phase if they have received the prescribed credits, have passed the qualifier, and have the agreement of a faculty member to be their PhD advisor.

2.3 Course Requirements

In order to obtain a PhD or MSc degree, a student has to satisfy the following coursework requirements (in addition to the general Jacobs University requirements):

1. For a PhD degree: graduate courses and seminars worth at least 95 ECTS credits

2. For a Master's degree: graduate courses and seminars worth at least 95 ECTS credits

3. For both degrees: the courses Introductory Algebra (100 321) or Algebra (100 421), Real Analysis (100 313), and Introductory Complex Analysis (100 312) or Topics in Complex Analysis (100 412).

Throughout their studies, graduate students are required to take one graduate level seminar each semester. In addition, all graduate students are expected to regularly attend the mathematics colloquium.

Graduate classes are 400 level courses and above; up to three elective 300 level undergraduate courses may be counted towards the course credit for PhD or Master's degrees.

Courses at 300 and 400 level carry 7.5 ECTS credits; advanced graduate seminars carry 5 ETCS credits. A research proposal, including presentation, carries 25 ECTS credits. The Master's thesis carries 25 ECTS credits as well.

Jacobs University Bremen reserves the right to substitute courses by replacements and/or reduce the number of mandatories/mandatory elective courses offered.

2.4 Qualifying Examination

Every graduate student working towards a PhD in mathematics must pass a comprehensive examination in mathematics at the beginning of the fourth semester. By the end of the third semester, the student has to request from the coordinator of the graduate program in mathematical sciences time and date for her or his examination.

The purpose of this examination is to manifest solid knowledge of advanced but core material in mathematics and/or applied and computational mathematics, to show the ability to make connections between areas of mathematics usually taught in different courses, and to demonstrate the potential for research in the mathematical sciences.

Students are free to choose between two examination formats, Mathematics or Applied and Computational Mathematics.

Both examination formats are oral, the exam takes at least 90 minutes. The material covered is described below according to format.

Mathematics

1. Algebra
2. Real Analysis
3. Complex Analysis

plus a choice of one among the following topics

4. Topology
5. Numerical Analysis
6. Partial Differential Equations
7. Mathematical Physics
8. Functional Analysis
9. Probability Theory

Applied and Computational Mathematics

1. Concepts from Algebra and Complex Analysis
2. Real Analysis
3. Numerical Analysis

plus a choice of one among the following topics

4. Scientific Computing
5. Partial Differential Equations
6. Mathematical Physics

7. Functional Analysis
8. Probability Theory

The syllabi for all topics are presented in Section 4. The examination is given by three professors appointed by the graduate committee.

2.5 Master's Option

Any graduate student may at any time request to work for a Master's degree, independently of whether or not he or she continues to work for a PhD degree.

The graduation requirements are specified by the Jacobs University's policies; the coursework requirements are described in Section 2.3. There is no separate Master's examination or qualifying examination.

The following table describes a study plan for a graduate student who has entered the program with a Bachelor's degree and wishes to conclude his or her graduate education with a Master's degree after four semesters.

Semester	Coursework	Research
1–2	3 courses and 1 seminar	
3	3 courses and 1 seminar	Preliminary work
4	1 course and 1 seminar	Master's thesis

3 Courses

The graduate courses offered in the first three semesters of the graduate program in Mathematical Sciences have two purposes. The first is to provide a solid and broad foundation of mathematical knowledge that is needed for doing research in mathematical sciences. The second purpose (at least of some of the courses) is to also provide an introduction to a specific area of current research, so as to give the students some basis for their decision on their area of specialization.

3.1 300 Level Courses

Depending on their preparation, students may decide to enroll in a certain number of 300 level courses that provide the necessary pre-requisites for many 400 level graduate courses and also cover substantial portions of the qualifying examination syllabi (see Section 4 below). For example, the courses

Introductory Complex Analysis
Real Analysis
Introductory Algebra
Manifolds and Topology
Stochastic Processes
Numerical Analysis

cover important aspects of a solid graduate education in Mathematical Sciences. The syllabi of 300 level courses can be found in the handbooks of the undergraduate majors Mathematics (100xxx) and Applied and Computational Mathematics (110xxx). Students should consult with their initial academic advisers on their choice of 300 level courses.

3.2 400 Level Courses

The list of 400 level gradute courses includes

Topics in Complex Analysis
Algebra
Advanced Algebra
Algebraic Geometry
Algebraic Topology
Differential Geometry
Lie Groups and Lie Algebras
Modern Geometry
Dynamical Systems
Functional Analysis
Partial Differential Equations
Topics in Applied Analysis

3.3 Reading Courses in Mathematics

Specialized topics, often related to faculty research areas, are taught in the form of reading courses. Offers depend on student and faculty interests. As a rule, reading courses carry 5 credits. Usage of these credits towards a graduate degree requires approval by the graduate education committee.

3.4 Graduate Seminars

Graduate Seminars include

Mathematics Colloquium
Graduate Research Seminar

In addition, there are regular research seminars run by the faculty of the graduate program on advanced subjects and/or on topics of current research interests. Such seminars carry 5 ECTS each; as a rule, participants receive Pass/Fail grades.

4 Qualifying Examination Syllabi

4.1 Algebra

1. **Groups:** monoids, groups, cyclic groups, normal subgroups, operation of a group on a set, Sylow subgroups, direct sums and free abelian groups, finitely generated abelian groups, nilpotent and solvable groups, Jordan-Hölder's theorem.

2. **Rings:** rings and homomorphisms, ideals, commutative rings, localization, principal rings, PIDs, Euclidean rings, UFD's.
3. **Polynomials:** Euclidean algorithm, unique factorization in several variables, Hilbert's Nullstellensatz, criteria for irreducibility, the derivative and multiple roots, symmetric polynomials.
4. **Modules:** homomorphisms, exact sequences, direct sums of modules, free modules, projective and injective modules, modules over PIDs, tensor product of modules.
5. **Noetherian rings and modules:** basic criteria, Hilbert's basis theorem, power series.
6. **Fields and Galois theory:** finite and algebraic extensions, algebraic closure, splitting fields and normal extensions, separable and Galois extensions, main theorem of Galois theory, finite fields, primitive elements, purely inseparable extensions (at least know one example), the norm and trace, cyclic extensions, solvable and radical extensions. (Including the standard applications: regular n-gons, trisecting an angle, . . .).

Main reference

T.W. Hungerford, "Algebra," Springer-Verlag.

Alternative and complementary reading

S. Lang, "Algebra," Springer-Verlag.

4.2 Real Analysis

1. **Measures:** algebras, sigma algebras, measures, outer measures, premeasures, continuation and completion, Borel and Lebesgue measures on $\mathbb{R}^d$.
2. **Integration:** measurable functions, integrability of real and complex valued functions, limit theorems, modes of convergence, product measures and Fubini-Tonelli theorem, the Lebesgue integral on $\mathbb{R}^d$.
3. **Signed measures:** Hahn and Jordan decomposition theorems, absolute continuity, Radon Nikodym theorem, bounded variation.
4. L^p **spaces:** basic inequalities (Hoelder, Minkowski, Jensen, Chebyshev), bounded linear functionals on L^p, Hardy-Littlewood maximal function.
5. **Elements of functional analysis:** Hilbert spaces, complete orthonormal systems, Fourier series on $\mathbb{T}^d$, Fourier transform on $\mathbb{R}^d$.
6. **Elements of distribution theory:** $D(\mathbb{R}^d)$ and $D'(\mathbb{R}^d)$, Schwarz class and tempered distributions, basic operations on distributions.

Main reference

G. Folland, "Real Analysis: Modern Techniques and Their Applications," second edition, Wiley.

Alternative and complementary reading

W. Rudin, "Real and Complex Analysis," third edition, Mc Graw-Hill.
E.H. Lieb and M. Loss, "Analysis," second edition, AMS, 2001 (Chapters 1–6).
H.L. Royden, "Real Analysis," third edition, Macmillan.

4.3 Complex Analysis

The material covered in

L. Ahlfors, "Complex Analysis," third edition, McGraw-Hill.

4.4 Concepts from Algebra and Complex Analysis

This qualifier covers the fundamental concepts from the Algebra and Complex Analysis qualifiers.

Concepts from Algebra

1. **Groups:** monoids, groups, cyclic groups, normal subgroups, operation of a group on a set, direct sums and free abelian groups, finitely generated abelian groups.
2. **Rings:** rings and homomorphisms, ideals, commutative rings, principal rings, PIDs, Euclidean rings, UFDs.
3. **Polynomials:** Euclidean algorithm, unique factorization in several variables.
4. **Fields and Galois theory:** finite and algebraic extensions, algebraic closure, splitting fields and normal extensions, separable and Galois extensions, main theorem of Galois theory, finite fields.

Concepts from Complex Analysis The material covered in Chapters 1–4 and Sections 1–2 of Chapter 5 in

L. Ahlfors, "Complex Analysis," third edition, McGraw-Hill.

Main reference

T.W. Hungerford, "Algebra," Springer-Verlag.
L. Ahlfors, "Complex Analysis," third edition, McGraw-Hill.

Alternative and complementary reading

S. Lang, "Algebra," Springer-Verlag.
J.B. Conway, "Functions of one complex variable," second edition, Springer-Verlag.

4.5 Topology

The material covered in

K. Jänich, "Topology," Springer-Verlag.

4.6 Numerical Analysis

1. **Basics of error, computational work, and condition/stability analysis.**
2. **Numerical linear algebra:** direct (Gauss/LU/QR) and iterative solvers (GS, ILU, PCG, GMRES), linear least-squares problems, eigenvalue solvers.
3. **Numerical methods of analysis:** integration, interpolation and approximation, FFT.
4. **Nonlinear equations and optimization:** Newton-type methods, nonlinear least-squares methods, global search methods, unconstrained and constrained smooth optimization.
5. **Initial value problems for ODEs:** implicit and explicit one- and multi-step methods, extrapolation methods, convergence and stability concepts, stiff problems.
6. **PDE discretization:** finite difference and finite element method for elliptic and parabolic PDE, solution of sparse linear systems.

Main reference

A. Quarteroni, R. Sacco, and F. Saleri, "Numerical Mathematics," Springer-Verlag.

Alternative and complementary reading

G. Golub and C. van Loan, "Matrix Computations," third edition, Johns Hopkins Press.
E. Hairer, S.P. Norsett, and G. Wanner, "Solving Ordinary Differential Equations I and II," Springer-Verlag.
S. Larsson and V. Thomee, "Partial Differential Equations with Numerical Methods," Springer-Verlag.
J. Nocedal and S.J. Wright, "Numerical Optimization," second edition, Springer-Verlag.

4.7 Partial Differential Equations

1. **Linear model equations:** Transport, Laplace, Heat, Wave Equations (classical solution techniques, representation formulas, energy methods).
2. **First-order Nonlinear Equations:** method of characteristics, introduction to Hamilton-Jacobi equations and conservation laws.
3. **Sobolev spaces and elliptic boundary value problems:** existence and regularity of weak solutions, weak and strong maximum principles.
4. **Linear evolution equations:** weak solutions of parabolic and hyperbolic equations, semi-group techniques.
5. **Variational and Nonvariational Methods for Nonlinear PDE:** existence and regularity of minimizers and critical points of energy functionals associated with PDE.

Main references

L.C. Evans, "Partial differential equations," AMS. (Part I, II and Selected Topics from Part III.)

Alternative and complementary reading

J. Jost, "Partial differential equations," Springer-Verlag, 2002.

4.8 Mathematical Physics

Syllabus offered by Professors Schupp and Huckleberry upon request.

4.9 Functional Analysis

1. **Topological vector spaces and completeness:** Baire category theorem, open mapping theorem, closed graph theorem.

2. **Convexity, weak topologies, duality in Banach spaces and compact operators:** Hahn-Banach theorem, Banach-Alaoglu theorem, Krein-Milman theorem.

3. **Distributions, Fourier analysis and applications to differential equations:** Haar measure on compact groups, Fourier analysis on Groups, Fourier transforms, Fourier series, distributions and tempered distributions.

4. **Banach algebras and spectral theory:** Gelfand-Mazur theorem, commutative Banach algebras, Gelfand transforms, bounded operators on a Hilbert space, spectral theorem for normal and bounded operators.

Main references

W. Rudin, "Functional Analysis," McGraw-Hill.
J.B. Conway, "A Course in Functional Analysis," second edition, Springer-Verlag.

Alternative and complementary reading

H. Brezis, "Functional Analysis, Sobolev Spaces and Partial Differential Equations," Springer-Verlag.
G. Folland, "Real Analysis," Wiley.
W. Rudin, "Fourier Analysis on Groups," Wiley. (Pages 1–13.)

4.10 Probability Theory

1. **General probability spaces:** Discrete and geometric probabilities.

2. **Random variables:** Joint distribution function, density (if exists), quantiles. Examples, discrete and continuous (uniform, exponential, normal).

3. **One-dimensional transformations of distributions.**

4. **Expectation of a random variable (and its function):** Variance, moments.

5. **Joint distributions and independence:** marginal distributions, joint density.

6. **Infinite sequences of random variables:** Borel-Cantelli lemma. Modes of convergence. Convergence of expectations. Weak and strong laws of large numbers, central limit theorem.

7. **Joint distributions:** conditioning, correlation, and transformations. Conditional distributions, conditional expectation. Total probability formula (continuous case). Regression and correlation. Multidimensional transformations of distributions. Distribution and density of sum, product and quotient of one-dimensional random variables.

8. **Countable Markov chains, Random walks.**

9. **Classification of finite Markov chains.**

10. **Brownian motion:** Construction and basic properties.

Main references

L.B. Koralov and Y.G. Sinai, “Theory of Probability and Random Processes,” Springer-Verlag.

Alternative and complementary reading

A.N. Shiryaev, “Probability,” second edition, Springer-Verlag.

4.11 Scientific Computing

1. **Large scale scientific computing:** computational and analytical aspects of FEM, FDM, FVM, multigrid method, domain decomposition method, error estimators, adaptivity, solution of sparse systems.

2. **High performance computing:** principles of parallel and distributed computing, computation on graphics hardware, data structures, complexity and runtime efficiency of standard algorithms (FFT, dense and sparse solvers for numerical linear algebra)

3. **Application area, select one from:** (a) Modeling with PDEs, (b) Signal and Image Processing, (c) Scientific Visualization.

Main references

A. Quarteroni, R. Sacco, and F. Saleri, “Numerical Mathematics,” Springer-Verlag.
S. Larsson and V. Thomee, “Partial Differential Equations with Numerical Methods,” Springer-Verlag.
A.M. Bruaset and A. Tveito, “Numerical solution of partial differential equations on parallel computers,” Springer-Verlag.

Alternative and complementary reading

S. Salsa, "Partial Differential equations in action: from modeling to theory," Springer-Verlag.
G. Aubert and P. Kornprobst, "Mathematical Problems in Image Processing – Partial Differential Equations and the Calculus of Variations," Springer-Verlag.

5 Faculty Research Areas

Complex geometry Prof. Alan Huckleberry

Group theory and geometry Dr. Keivan Mallahi-Karai

Affine algebraic geometry, commutative algebra, cryptography Dr. Stefan Maubach

Quasiconformal uniformization and dynamics Prof. Daniel Meyer

Partial differential equations and fluid dynamics Prof. Marcel Oliver

Approximation theory, numerical analysis, multiscale methods Prof. Peter Oswald

Algebra, geometry, Lie theory, and representation theory Prof. Ivan Penkov

Modeling of bio-medical processes, partial differential equations Prof. Tobias Preusser

Applied harmonic and functional analysis, wavelets, Gabor analysis Prof. Götz E. Pfander

Dynamical systems, conformal and fractal geometry Prof. Dierk Schleicher

Mathematical and theoretical physics Prof. Peter Schupp

Mathematical physics, Riemannian geometry, wavelet theory Prof. Raymond O. Wells

6 Course Descriptions

6.1 300 Level Courses

100313: Real Analysis

Short Name: RealAna
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents Real Analysis is one of the core advanced courses in the Mathematics curriculum. It introduces measures, integration, elements from functional analysis, and the theory of function spaces. Knowledge of these topics, especially Lebesgue integration, is instrumental in many areas, in particular, for stochastic processes, partial differential equations, applied and harmonic analysis, and is a prerequisite for the graduate course in Functional Analysis.

The course is suitable for undergraduate students who have taken Analysis I/II, and Linear Algebra I; it should also be taken by incoming students of the Graduate Program in the Mathematical Sciences. Due to the central role of integration in the applied sciences, this course provides an excellent foundation for mathematically advanced students from physics and engineering.

100312: Introductory Complex Analysis

Short Name:	ComplexAnal
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100212 and 100221
Corequisites:	None
Tutorial:	Yes

Course contents This course introduces the theory of functions of one complex variable. It centers around the notion of complex differentiability and its various equivalent characterizations. Unlike differentiability for real functions, complex differentiability is a very strong property; for example it implies that the function is differentiable infinitely often and that it is represented by its Taylor series in a neighborhood of every point in its domain of definition. This results in a very nice and elegant theory that is used in many areas of mathematics.

Topics include holomorphic functions, Cauchy integral theorem and formula, Liouville's theorem, fundamental theorem of algebra, isolated singularities and Laurent series, analytic continuation and monodromy theorem, residue theorem, normal families and Montel's theorem, and the Riemann mapping theorem.

Possible further topics are elliptic and modular functions, the Riemann zeta function, introduction to Riemann surfaces.

100321: Introductory Algebra

Short Name:	IntroAlgebra
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100221
Corequisites:	None
Tutorial:	Yes

Course contents This course gives an introduction to three basic types of algebraic structures: groups, commutative rings, and fields. (If time permits, a fourth one: modules.) Here is a more detailed list of topics to be covered.

Group Theory: Definitions and key examples. Cosets and Lagrange's theorem. Group homomorphisms and basic constructions including quotient groups, direct and semi-direct products. Some examples of (important) groups. Group actions and orbit-stabilizer theorem. Possibly: Sylow theorems.

(Commutative) Rings: Definitions and elementary properties. Ideals, ring homomorphisms and quotient rings. Domains, Euclidean domains, principal ideal domains and unique factorization. Polynomial rings.

Field extensions: Roots of polynomials. Irreducibiliy criteria. Finite and algebraic field extensions. Finite fields. Possibly: Splitting fields and algebraic closure. Constructions with straightedge and compass.

If time permits *Modules:* Definitions and basic constructions. Linear maps and exact sequences. Direct products and sums. Structure theory for finitely generated modules over a principal ideal domain.

100331: Introductory Number Theory

Short Name:	IntroNumTheory
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100211 and 100321
Corequisites:	None
Tutorial:	Yes

Course contents This course gives a rst introduction to number theory. It starts with Elementary Number Theory, covering topics such as congruences, the Chinese Remainder Theorem, Fermat's Little Theorem and Euler's extension; these have interesting applications to cryptography (such as the famous RSA algorithm). Further topics include Gaussian integers, quadratic reciprocity, Diophantine equations, Minkowski's lattice point theorem, as well as sums of two, three, and four squares.

The course will then move on beyond elementary number theory. Depending on the interests of students and instructor, possible topics are Pell's equation and continued fractions, the Prime Number Theorem, Dirichlet's theorem about primes in arithmetic progressions, or elliptic curves.

100332: Discrete Mathematics

Short Name:	DiscMath
Type:	Lecture
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	Yes

Course contents This course is open to anyone with interest and some experience in mathematics (for others, *General Mathematics and Computational Science I* and/or *General Mathematics and Computational Science II* is recommended).

Discrete mathematics is a branch of mathematics that deals with discrete objects and has naturally many applications to computer science. This course introduces the basics of the subject, in particular (enumerative) combinatorics, graph theory, as well as mathematical logic.

Enumerative combinatorics includes the binomial and multinomial coefficients, the pigeonhole principle, the inclusion-exclusion formula, generating functions, partitions, and Young diagrams.

Fundamental topics in graph theory include trees (spanning trees, enumeration of trees), cycles (Eulerian and Hamiltonian cycles), planar graphs (Kuratowski's theorem), colorings, and matching (perfect matchings, Hall's theorem).

In mathematical logic, among the basic topics are the Zermelo-Fraenkel axioms, as well as cardinal and ordinal numbers and their properties.

Additional topics may be chosen depending on interests of instructor and students.

100341: Introductory Topology

Short Name: IntroTopology
Type: Lecture
Credit Points: 7.5
Prerequisites: 100211 and 100221
Corequisites: None
Tutorial: Yes

Course contents This course is an introduction to some basic concepts and techniques in topology. The first part of the course builds on material from *Analysis I*, in particular the topology of metric spaces. We introduce topological spaces and continuous maps and proceed to discuss properties of spaces including connectedness, compactness and the Hausdorff property. Basic constructions such as the product and quotient of spaces are also treated.

The second part of the course deals with basic concepts of algebraic topology. We introduce the notion of homotopy, construct the fundamental group of a space and introduce the Seifert–van Kampen theorem, a key tool for computing fundamental groups. We discuss covering spaces and their relation with the fundamental group, including the construction of the universal covering space.

The course concludes with a basic treatment of homology groups and their properties, which are a fundamental tool for distinguishing topological spaces and mappings between them.

100353: Manifolds and Topology

Short Name: ManifoldsTop
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212 and 100221
Corequisites: None
Tutorial: Yes

Course contents This course is an introduction to the language and some of the fundamental concepts of modern geometry. Manifolds are among the most fundamental concepts of mathematics: curves and surfaces are important special cases that have historical significance.

The course starts with introducing the notion of a manifold, followed by examples that naturally arise in various areas of mathematics. Differentiability, tangent spaces and vector fields are then defined. This will be followed by establishing the notion of integration on manifolds. We will then formulate and prove Stokes' theorem, which is the higher-dimensional generalization of the fundamental theorem of calculus. Among the further topics that are discussed in the course are: orientation, degree of a map, Lie groups and their actions. The classification of one- and two-dimensional manifolds and the Poincaré-Hopf theorem will be some of the highlights of the course.

100361: Ordinary Differential Equations and Dynamical Systems

Short Name: DynSystems
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212 and 100221
Corequisites: None
Tutorial: Yes

Course contents Dynamical systems is an topic which links pure mathematics with applications in physics, biology, electrical engineering, and others. The course will furnish a systematic introduction to ordinary differential equations in one and several variables, focusing more on qualitative aspects of solutions than on explicit solution formulas in those few cases where such exist. It will be shown how simple differential equations can lead to complicated and interesting, often "chaotic" dynamical behavior, and that such arise naturally in the "real world". We will also discuss time-discrete dynamical systems (iteration theory) with its relations and differences to differential equations.

100362: Introductory Partial Differential Equations

Short Name: Intro PDE
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents This course is a rigorous, but elementary introduction to the theory of partial differential equations: classification of PDEs, linear prototypes (transport equation, Poisson equation, heat equation, wave equation); functional setting, function spaces, variational methods, weak and strong solutions; first order nonlinear PDEs, introduction to conservation laws; exact solution techniques, transform methods, power series solutions, asymptotics.

This course alternates with *Partial Differential Equations* which takes a functional analytic approach to partial differential equations.

100382: Stochastic Processes

Short Name: StochProc
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents This course is an introduction to the theory of stochastic processes. The course will start with a brief review of probability theory including probability spaces, random variables, independence, conditional probability, and expectation.

The main part of the course is devoted to studying important classes of discrete and continuous time stochastic processes. In the discrete time case, topics include sequences of independent random variables, large deviation theory, Markov chains (in particular random walks on graphs), branching processes, and optimal stopping times. In the continuous time case, Poisson processes, Wiener processes (Brownian motion) and some related processes will be discussed.

This course alternates with *Applied Stochastic Processes*.

100383: Applied Stochastic Processes

Short Name: ApplStochProc
Type: Lecture
Credit Points: 7.5
Prerequisites: 100212
Corequisites: None
Tutorial: Yes

Course contents This course aims at an introduction to the mathematical theory of financial markets that discusses important theoretical concepts from the theory of stochastic processes developed in parallel to their application to the mathematical finance.

The applied part of this course revolves around the central question of option pricing in markets without arbitrage which will be first posed and fully solved in the case of binomial model. Interestingly enough, many of the fundamental concepts of financial mathematics such as arbitrage, martingale measure, replication and hedging will manifest themselves, even in this simple model. After discussing conditional expectation and martingales, more sophisticated models will be introduced that involve multiple assets and several trading dates. After discussing the fundamental theorem of asset pricing in the discrete case, the course will turn to continuous processes. The Wiener process, Itō integrals, basic stochastic calculus, combined with the main applied counterpart, the Black-Scholes model, will conclude the course.

This course alternates with *Stochastic Processes*.

100391: Guided Research Mathematics I

Short Name:	GR Math I
Type:	Self Study
Credit Points:	7.5
Prerequisites:	permission of instructor
Corequisites:	None
Tutorial:	No

Course contents Guided Research allows study, typically in the form of a research project, in a particular area of specialization that is not offered by regularly scheduled courses. Each participant must find a member of the faculty as a supervisor, and arrange to work with him or her in a small study group or on a one-on-one basis.

Guided research has three major components: Literature study, research project, and seminar presentation. The relative weight of each will vary according to topic area, the level of preparedness of the participant(s), and the number of students in the study group. Possible research tasks include formulating and proving a conjecture, proving a known theorem in a novel way, investigating a mathematical problem by computer experiments, or studying a problem of practical importance using mathematical methods.

Third year students in Mathematics and ACM are advised to take 1–2 semesters of Guided Research. The Guided Research report in the spring semester will typically be the Bachelor's Thesis which is a graduation requirement for every Jacobs University undergraduate. Note that the Bachelor's Thesis may also be written as part of any other course by arrangement with the respective instructor of record.

Students are responsible for finding a member of the faculty as a supervisor and report the name of the supervisor and the project title to the instructor of record no later than the end of Week 4. A semester plan is due by the end of Week 6.

100392: Guided Research Mathematics and BSc Thesis II

Short Name:	GR Math II
Type:	Self Study
Credit Points:	7.5
Prerequisites:	permission of instructor
Corequisites:	None
Tutorial:	No

Course contents As for *Guided Research Mathematics I*.

110341: Numerical Analysis

Short Name:	NumAnal
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100212
Corequisites:	None
Tutorial:	Yes

Course contents This course an advanced introduction to Numerical Analysis. It complements *ESM 4A – Numerical Methods*, placing emphasis, on the one hand, on the analysis of numerical schemes, on the other hand, focusing on problems faced in large-scale computations. Topics include sparse matrix linear algebra, large scale and/or stiff systems of ordinary differential equations, and a first introduction to methods for partial differential equations.

110361: Mathematical Modeling in Biomedical Applications

Short Name:	MathMod BioMed
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100212
Corequisites:	None
Tutorial:	Yes

Course contents The course discusses the area of mathematical modeling in biomedical applications. It includes an introduction into the basic principles of mathematical modeling, and it covers a variety of models for growth and treatment of cancer with increasing complexity ranging from simple ordinary differential equations to more complicated free boundary problems and partial differential equations. Further models for the description of physiology in the human body like blood flow and breathing are briefly touched as well.

6.2 400 Level Courses

100412: Topics in Complex Analysis

Short Name:	CompAnalysis
Type:	Lecture
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents Topics in Complex Analysis builds on the material taught in the undergraduate Complex Variables course. After a quick review of the most important results and concepts, some more advanced topics are covered. Possible subjects are Riemann Surfaces, Elliptic Functions and Modular Forms, Complex Dynamics, Geometric Complex Analysis, or Several Complex Variables. Which subjects are chosen will depend on the instructor and on the students' interests. This course may also provide an introduction to a specific area of research, leading to possible PhD thesis projects.

Due to the varying content, this course can be taken multiple times for credit.

100421: Algebra

Short Name: Algebra
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Advanced topics from algebra, including groups, rings, ideals, fields, and modules, continuing the course *Introductory Algebra*.

100422: Advanced Algebra

Short Name: AdvAlg
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course develops more advanced topics in algebra beyond those from *Algebra*, including Galois theory, commutative algebra and its relation to algebraic geometry, as well as elements of noncommutative algebra.

100423: Algebraic Geometry

Short Name: AlgGeometry
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Algebraic geometry is the study of geometry using algebraic tools: the geometric objects are the common roots of a set of polynomials in several variables. Many geometric properties can be studied in terms of algebraic properties of these polynomials, using the powerful machinery of algebra to study geometry.

Basic concepts from *Algebra* and *Introductory Algebra* are used in this course. Among the studied subjects are affine and projetive varieties, schemes, curves, and cohomology.

100442: Algebraic Topology

Short Name: AlgebrTopology
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course is mostly concerned with the comprehensive treatment of the fundamental ideas of singular homology/cohomology theory and duality. The knowledge of fundamental concepts of algebra as well as of general topology is assumed (at a level of *Introductory Topology* and *Introductory Algebra*.

The first part studies the definition of homology and the properties that lead to the axiomatic characterization of homology theory. Then further algebraic concepts such as cohomology and the multiplicative structure in cohomology are introduced. In the last section the duality between homology and cohomology of manifolds is studied and few basic elements of obstruction theory are discussed.

The graduate algebraic topology course gives a solid introduction to fundamental ideas and results that are used nowadays in most areas of pure mathematics and theoretical physics.

100451: Differential Geometry

Short Name: DiffGeom
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Differential geometry is the study of differentiable manifolds. Assuming basic concepts such as manifolds, differential forms, and Stokes' theorem, the focus in this course is on Riemannian geometry: the study of curved spaces which is at the heart of much current mathematics as well as mathematical physics (for example, General Relativity).

100452: Lie Groups and Lie Algebras

Short Name: LieGroups
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents A Lie group is a group with a differentiable structure, the tangent space at the identity element of a Lie group is its Lie algebra. Lie groups and Lie algebras are indispensable in many areas of mathematics and physics. As a mathematical subject on its own, Lie theory has led to many beautiful results, such as the famous classification of semisimple Lie algebras. In physics, Lie groups and their representations are essential to the theory of elementary particles and its current developments. Due to the close correspondence of physical phenomena and abstract mathematical structures, the theory of Lie groups has become a showcase of mathematical physics.

The course presents fundamental concepts, methods and results of Lie theory and representation theory. It covers the relation between Lie groups and Lie algebras, structure theory of Lie algebras, classification of semisimple Lie algebras, finite-dimensional representations of Lie algebras, and tensor representations and their irreducible decompositions.

A solid background in multivariable real analysis and linear algebra is presumed. Familiarity with some basic algebra and group theory will also be helpful. No prior knowledge of differential geometry is necessary.

100453: Modern Geometry

Short Name: ModernGeometry
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents The course serves as an introduction, at the advanced level, to the basic concepts of modern geometry.

The following concepts, known from the 300-level courses, should be briefly reviewed: concept of a manifold, the simplest examples of manifolds, and the concept of homotopy.

The core of the course will consist of explaining material related to the following topics: Lie groups, homogeneous spaces, symmetric spaces, fiber bundles, vector bundles, Morse theory, differential topology of mappings and submanifolds.

This material will provide a solid background for the 400-level courses, *Differential Geometry* and *Algebraic Topology*.

100461: Dynamical Systems

Short Name: DynSystems
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Dynamical systems is the study of the long-term behavior of anything in motion. The classical motivating topic is the stability of the solar system or, more recently, the study of weather prediction.

One theme in the course is the study of the underlying questions and difficulties in terms of model equations that are much simpler, often 1-, 2-, or at most 3-dimensional, but yet show rich and interesting dynamical features. A fundamental tool is to describe the dynamics of flows in terms of iterated maps of lower dimension, which are of great interest in their own right. Among the topics covered are circle homeomorphisms and endomorphisms, including rotation numbers, the quadratic family, toral automorphisms, horseshoes and the solenoid, the Lorenz systems, symbolic dynamics and shifts, and Sharkovski's theorem.

A second topic are ways to describe and quantify how complicated dynamical systems are: recurrence, topological transitivity and periodic orbits, mixing dynamics, topological and metric entropy, Lyapunov exponents, ergodicity and Birkhoff's theorem, and more.

Finally, there will be a discussion of general hyperbolic dynamics, including the stable/unstable manifold theorem and the shadowing lemma (not necessarily with detailed proofs in full generality).

100471: Functional Analysis

Short Name:	FunctAnalysis
Type:	Lecture
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents This course assumes basic knowledge of measure and integration theory, and of classical Banach and Hilbert spaces of measurable functions. Functional Analysis focuses on the description, analysis, and representation of linear functionals and operators defined on general topological vector spaces, most prominently on abstract Banach and Hilbert spaces.

Even though abstract in nature, the tools of Functional Analysis play a central role in applied mathematics, e.g., in partial differential equations. To illustrate this strength of Functional Analysis is one of the goals of this course.

100472: Partial Differential Equations

Short Name:	PDE
Type:	Lecture
Credit Points:	7.5
Prerequisites:	100313
Corequisites:	None
Tutorial:	No

Course contents The course is an introduction to the theory of partial differential equations in a Sobolev space setting. Topics include Sobolev spaces, second order elliptic equations, parabolic equations, semi-groups, and a selection of nonlinear problems.

This course differs from the approach taken in *Introductory Partial Differential Equations* which focuses on solutions in classical function spaces via Greens functions. It may therefore be taken by students who have attended *Introductory Partial Differential Equations*, but we will again start from basic principles so that *Introductory Partial Differential Equations* is not a prerequisite.

110411: Topics in Applied Analysis

Short Name: ApplAnalysis
Type: Lecture
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents The course Topics in Applied Analysis introduces to a variety of fundamental analytic tools and methods used in the theory, modeling, and numerical simulation of phenomena in the natural sciences. The course is offered with different contents in different years, the choice will depend on the instructor. Examples of areas currently covered are applied harmonic analysis and operator theory, perturbation theory and asymptotic analysis, approximation theory, and others. Students specializing in applied mathematics or applied sciences may participate in this course more than once.

6.3 Graduate Seminars

100591: Mathematics Colloquium

Short Name: MathColloquium
Type: Seminar
Credit Points: None
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents The weekly mathematics colloquium features talks by international scientists for the entire mathematical community, broadening horizons and encouraging formal or informal interactions.

100491: Graduate Research Seminar

Short Name: GradResearchSem
Type: Seminar
Credit Points: 5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course is intended for beginning graduate students to help them identify interesting areas of research and possible thesis subjects and advisors. It consists of lectures mainly by professors, but also by other faculty, about current areas of research in mathemaical sciences, with particular emphasis on research areas of Jacobs faculty. Students get involved in discussions of all the areas of research; during the course of the semester, they choose at least three topics which they investigate further and which they elaborate into a research report. At the end of the semester, every student presents at least one of these reports. Participation is also open for advanced undergraduates looking for topics for their undergraduate theses, the results of which are presented as well.

Graduate research seminar participants receive a grade on the scale 1.0 through 5.0.